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ABSTRACT

Systems Analysis uses many disciplines and sophisticated techniques to determine the relative effectiveness of alternate systems. This relationship can then be reversed by an arbitrary choice of a discount rate. In the private sector, the discount rate is determined by the cost of capital. In the public sector, the cost of acquiring capital is not clearly defined. The following proposals for the interest rate in the public sector are considered: (1) the government bond rate; (2) the rate of growth of the national product; and (3) a rate derived from the average rate of return in the private sector. It is concluded that the interest rate in the public sector currently lies between 4.75 and 10 percent.

The circumstances which generate uncertainty and the means of handling this uncertainty are also discussed. A procedure is recommended which modifies the difference between expected costs to make them equally significant. Particular attention is focused on uncertainty occasioned by changing technology and the probability of war. It is concluded that a unique estimate of the risk component is indeterminate and that uncertainty should be considered in the context of the specific systems under consideration.

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CHAPTER I

INTRODUCTION

Systems analysis uses many disciplines and sophisticated techniques, often at considerable expense, to determine the relative effectiveness of alternate systems. The relationship so determined can then be reversed by an arbitrary choice of a discount rate. Such a situation arises when a proposed system, characterized by high development costs and relatively low operating costs, is compared with a system already developed but with higher operating costs. Assuming equal effectiveness, the savings in reduced operating costs can be thought of as an annuity purchased for an amount equal to the difference in initial costs. The decision rule is to accept the new system if the annuity is greater than the interest premium sacrificed by the additional investment. The criterion is clear. The problem is to evaluate the interest premium sacrificed. In the private sector, net returns are discounted using the cost of capital or the bank interest rate. In the public sector, funds allocated by the annual budget cannot be similarly invested to draw interest. Although there is no interest rate in the public sector corresponding to the market rate of interest, there is an opportunity cost for public investments. First, there is the actual cost of acquiring capital and secondly, there may be a difference in returns to the economy as a whole due to the transfer of resources away from the private sector. The cost of acquiring capital and the marginal rate of return on capital are not clearly defined for the public sector. Consequently, the discount rate for a non-market context has not yet been determined.

The discount rate is actually composed of two elements, a normative component and a risk component. The former reflects the opportunity cost of an investment under conditions of certainty. The latter expresses the degree of uncertainty of investment outcomes. Future costs and returns are stochastic elements whose variance must be considered. Some confusion exists in the literature when no distinction is made between these components or when one is discussed without consideration of the other. For example, some authors feel that the risk component is indeterminate and therefore reject the entire concept of a discount rate in the non-market context. There is also some confusion as to the precise meaning of risk in such a context.

Most of the controversy surrounding the choice of an appropriate discount rate can be focused at two poles corresponding to the normative and risk components of the discount rate. There are a number of candidates for the former component. Among these are: (1) the government bond rate; (2) the rate of growth of the national product; and (3) a rate derived from the average rate of return in the private sector. There are also several schools of thought regarding the handling of risk. Some feel that a risk component should be included in the discount rate. Others feel that uncertainty should be dealt with explicitly at the level where costs are estimated. Finally, there are those who contend that the government should not worry about risk aversion at all. Chapter II addresses the problem of opportunity costs (normative rate) and Chapter III deals with the question of uncertainty (risk component). Chapter IV summarizes the results and discusses the collective treatment of the normative and risk components.

CHAPTER II

OPPORTUNITY COSTS

The interest rate

One concept of a discount rate is that it reduces a cost stream, that is, a vector of costs incurred over several time periods, to a scalar quantity in a single time frame. The discount rate, under conditions of certainty, equates each future cost in year t to an equivalent present value. It is based on an interest rate i which is the premium for deferring present consumption (not undertaking the investment) and is expressed as $(1 + i)^{-t}$. How do we evaluate this interest rate in a non-market context? Is it unique? If not, how do we estimate it? Even the market rate of interest is not unique. There are different interest rates for time deposits, personal loans, consumer credit, mutual funds, stocks, and bonds. In a very general sense, the interest rate is the cost of capital. One estimate of the cost of capital in the public sector is the *government bond rate*.

Another interpretation of the interest rate is that of opportunity cost. The underlying assumption is that, for any investment project, an alternate use of capital always exists. This alternate use of capital is the opportunity cost of the investment. One such alternative is the loaning of capital at the going interest rate. This interpretation is not suitable for the public sector because public funds cannot be loaned for this purpose. Another alternate use of capital might be a second investment project with different cost and benefit streams. In this sense, the discount rate is a criterion for allocating resources to one investment rather than to another.

This concept can be extended to the non-market context. In other words, the discount rate for public investment may be considered as the criterion for transferring resources between the private and public sectors.[1] Public investments utilize resources which could have been used by the private sector. The marginal rate of return in the private sector may be considered as the opportunity cost of a public investment. The exact discount rate to be used depends on the relative efficiency of capital in both sectors. The basic decision rule for determining the allocation of resources between alternate investment opportunities is that the marginal return in each must be equal. Accordingly, one proposal for the discount rate in the public sector is the *marginal rate of return in the private sector*.

Government purchase of goods and services are included in the computation of the GNP as are private expenditures. On the surface it would appear that an increase in government spending and a corresponding decrease in private spending produces the same effect on the economy. The total value of the GNP remains the same. However, the GNP is only an accounting device. It is descriptive rather than normative in that it tells how resources were used, not how they should have been used. The government may be hiring men to dig holes and to fill them in again in which case the GNP is an inflated representation of the true productivity of the economy. It may be assumed that the government is not engaged in aimless practices but is procuring

1. William Baumol, Statement, U.S. Congress, Joint Economic Committee, Hearings before Subcommittee on Economy in Government, 90th Congress, 1st Session on September 14, 19, 20, and 21, 1967 (Washington: Government Printing Office, 1967), p. 153.

and operating useful end items. Assuming further that a weapon system, purchased by the government, will yield the same level of utility as that achieved by a private expenditure of a similar amount of funds, future economic growth must still be considered. One weapon system will not produce another weapon system but part of the output in the private sector, which is curtailed by government expenditures, will be capital goods. Since these capital goods could have been used to create new growth, there is also an opportunity cost *in deferred growth* for most government purchases.

If a billion dollars worth of capital investment is curtailed this year, the opportunity cost is one billion dollars. If, however, the government expenditures are deferred until next year, the billion dollars could have been invested to yield a net return of x percent. This additional amount is still available after the government expenditures in the second year. Moreover, the net return continues to yield returns on itself in perpetuity. The opportunity cost is now one billion less these additional returns. Therefore, the two opportunity costs cannot be viewed as equal and the deferred cost stream must be discounted.

The investments of each government agency presumably are attempts to maximize national objectives. However, each individual agency may be suboptimizing with respect to its own objective functions. If there is no constraint placed upon the maximization of these individual objective functions, the national objectives may not be maximized at all. For example, excessive investment by the Department of Defense may have a retarding effect on the rate of economic growth. It may be wiser to defer some of these investments

until they can be better afforded. For this reason, the *rate of growth of the national product* was proposed as an estimate of the interest rate. This estimate may be regarded as a constraint to insure conformity with a primary national objective.

These three estimates of the interest rate, the bond rate, the rate of return in the private sector, and the rate of growth of the national product, were advanced for the following reasons.

(1) Resources are not free and there is a cost of acquiring capital.

(2) Public investments take resources away from the private sector.

(3) Curtailment of capital investment may deter future economic growth.

(4) With a rising GNP, deferred costs are borne more easily.

(5) Public investments should be constrained to insure conformity with national objectives.

All of these arguments imply that allowance for the opportunity cost of an investment should be made by discounting with some positive interest rate.

The bond rate

One estimate of the cost of capital in the public sector is the rate at which the government can borrow funds.[2] The government could alternatively refrain from borrowing and either pay off the national debt or lower taxes. It does borrow because it feels that the return on its investment at least equals the rate at which it is borrowing. In other words, the bond rate, currently evaluated at

2. Charles J. Hitch and Roland N. McKean, The Economics of Defense in the Nuclear Age (New York: Harvard University Press, 1963), p. 210.

4.75 percent, is the cost of capital. A counterargument is that the government does not borrow because it must but only to curtail present consumption. Proponents of this argument contend that the government has alternate ways of raising money such as taxation and printing money. Taxation is a direct means of transferring money from the private to the public sector. After taxes, the individual has less money with which to buy private consumption goods. These goods, or the resources to produce these goods, are now released for government use. Fiat money is another alternative. Merely by printing a sufficient quantity of new money, the government can bid the necessary resources away from the private sector. In the long run prices will rise until real income is the same as it would have been with taxation. Borrowing is the final alternative. The government can either sell bonds to banks and in effect create new money or it can sell to individuals. In the latter case, private consumption is voluntarily reduced by approximately the same amount as it would have been by taxation. Although individuals can now earn an additional income with interest premiums, prices will be bid upward as they were with fiat money and real income will again remain unchanged. In summary the government could finance its expenditures by printing money or by taxation with no clear indication of the interest rate. On the other hand, the government could raise all of its money by borrowing and the bond rate could be considered an estimate of capital cost. That the government must borrow is not a critical assumption to the use of the bond rate of interest.

Baumol has stated that the bond rate of interest is at least a lower limit on the estimate of the opportunity cost of resources used

by the public sector. He states that there are two alternate uses for these resources. They could be left in the hands of business firms or they could be left with individual consumers. In the former case he contends, and feels that most economists would agree, that these resources would yield a return easily above the bond rate of interest. To estimate the marginal return on consumer expenditures, he uses an interesting argument:

> For a consumer it is not quite so clear what the opportunity cost represents, because there is no discernible earning on resources utilized by a consumer for his own purposes. And yet one can find out what those earnings are by indirection. If a consumer chooses to invest some portion of his funds in Government securities, it is clear that the rate of return he gets for those funds for his own purposes must approximate the rate of return which the Government securities yield to him. Because if the rate of return from his own consumption were lower than what he can get for Government securities, clearly it would pay him to take even more of his money than he does and turn it into Government securities.[3]

This imputed rate of return assumes that consumers are optimizing in their behavior. Other actions by consumers show that this is not always the case. For example, the excessive rate of interest paid for installment credit certainly indicates otherwise. The low rate of return on insurance policies could also be cited as the marginal rate of return of consumer expenditures by the same argument. However, a distinction should be made between voluntary and constrained investments. Some consumers utilize credit because of an inability to save voluntarily. Others are simply unaware of implicit interest rates. An insurance policy represents the need to build an immediate estate without limiting present consumption too rigorously. The government should be regarded as a sophisticated consumer with financial

3. Baumol, op. cit., p. 151.

flexibility. In this sense, only voluntary consumer investments should be considered. There are other commercial rates of interest in this category. Among these are the rates paid by banks, credit unions, mutual funds and there are the average yields of stocks and corporate bonds. These interest rates are all higher than the government bond rate. The returns to business firms as pointed out by Baumol are also above the government bond rate. Therefore, this rate is accepted tentatively as a lower limit on the interest rate in the public sector.

Growth rate of the national product

An estimate for opportunity cost that goes below the bond rate is the rate of growth of the national product.[4] The essential argument is that the national product will increase over time. Therefore, the resources used this year are more valuable than those resources returned on the investment in future years. The latter are less valuable not because of a time preference but simply because they are part of a larger national product. Implicit in this argument is the concept that a decreasing marginal utility is proportional to increases in the national product. There is some intuitive appeal in this argument. Assuming a diminishing marginal efficiency of capital, the rate of growth of the national product is a decreasing function of the total level of investment. This level of investment also reflects the utility function of the society as a whole. The use of the growth rate as the interest rate will certainly determine the level of investment. If this rate is high, it will restrict public investments and

4. E. B. Berman, "The Normative Interest Rate," RAND Publication P-1796 (Santa Monice: The RAND Corporation, 1959), p. 22.

release more resources to the private sector; if it is low, it will generate an opposite effect. However, there is nothing to preclude an accelerating effect in one direction only. A low rate of growth would permit even more public investment and conceivably less capital investment. The result would be a lower rate of growth for the future. A non-optimal rate of growth should not be the basis of its own perpetuity and there is no guarantee that the current rate is in fact optimal. The relative efficiency of capital in the private and public sectors should be considered and an interest rate which permits the most efficient allocation of resources *in both sectors* should be chosen. For these reasons, the rate of growth of the national product will be rejected as an appropriate interest rate.

The rate of return in the private sector

Stockfisch advocates that the government interest rate be equal to the rate of return on investment in the private sector of the economy.[5] As stated previously, optimality will prevail when the marginal rate of return in the private sector equals the marginal rate of return in the public sector. There are, however, two problems with which to contend. First, it is extremely difficult, if not impossible, to measure in dollar terms the returns to most public investments. How, for example, can the effectiveness of a weapon system, the returns on manpower retraining, or the benefits of a space probe be evaluated in commensurable units? Secondly, the marginal rate of return in the private sector cannot be measured accurately because of present

5. J. A. Stockfisch, "The Interest Cost of Holding Military Inventory," Planning Research Corporation Report PRC R-156 (Los Angeles: Planning Research Corporation, 1960), p. 2.

accounting practices. Many firms invest in several different projects with varying rates of return, and only the *average* of these returns are reported for tax purposes. Theoretically, each firm would first invest in that project yielding the highest marginal return and invest successively in those projects with monotonically decreasing rates of return. The final rate of return realized for each firm should be equal to that of every other firm.

The ability to equate marginal returns implies a perfect mobility of capital. Even with this assumption, future returns are not deterministic. Many investments are based on incomplete knowledge, some yielding negative returns. The best that can be done at present is to arrive at an average rate of return for each firm by summing over all its investments and then to find the average of these rates by summing over all firms. Stockfisch has done this for 72 major corporations and determined empirically that the average rate of return was 16.5 percent.[6] (Hirshleifer, DeHaven, and Milliman made a similar study for utilities and arrived at a figure of 10 percent.[7]) Since there are rates of return above the average, the marginal rate of return is something less than this figure. It could conceivably be zero. However, if this were the case, firms would not borrow at the bank interest rate and even internal financing would be decreased to take advantage of the bank or bond interest rates. The point is that 16.5 percent is a maximum estimate for the interest rate and must be revised downward. Stockfisch himself has lowered his estimate to

6. Ibid., p. 14.

7. J. Hirshleifer, J. C. DeHaven, and J. W. Milliman, Water Supply: Economics, Technology and Policy (Chicago: University of Chicago Press, 1960), pp. 140-148.

the neighborhood of 10 to 15 percent. Baumol and Kamien also agree that this estimate should be closer to 10 percent.[8]

Although a precise estimate of the interest rate for the public sector appears to be impossible, upper and lower bounds have been placed on it. The rate of growth of the national product was rejected primarily because it did not distinguish between the private and public sectors and could not insure the optimal allocation of resources between these two sectors. The government bond rate of 4.75 percent was considered an acceptable estimate of the cost of capital in the public sector and this was the lowest estimate of the interest rate considered. An estimate of the marginal rate of return in the private sector, derived from an average rate of return, was also accepted as an indication of the opportunity cost of public investments. Although 15 percent is the maximum estimate, there appears to be more confidence in a figure of 10 percent as the maximum estimate. This upper limit could be refined by more precise reporting of business data. (A model for relating the average rate of return in the private sector to that in the public sector is presented in Appendix A.) Between the two limits of our estimate there is a spectrum of interest rates found within the private sector. These rates, however, reflect the cost of capital in the private sector based on varying degrees of risk. If we average the estimates of 4.75 and 10 percent, a figure of 7 or 8 percent seems appropriate. This is an arbitrary choice but one which minimizes the deviation from either estimate. Since public investments are often evaluated in comparison with other proposed investments, a sensitivity analysis, as outlined in Appendix B, over the range of the proposed interest rates may preclude the necessity for a precise estimate.

8. Baumol, op. cit., pp. 160-169.

CHAPTER III

UNCERTAINTY

Risk or uncertainty exists because of the stochastic nature of investments. In decision theory, risk is characterized by a known state space and a known probability distribution for the occurence of each state. In the case of uncertainty, the probability distribution is unknown. The two terms are used here interchangeably and they both denote the possibility of incurring cost streams greater or less than those estimated. This definition does not imply that future benefits or returns are deterministic. They are treated as such in the non-market context because of the measurability problem. Any evaluation of effectiveness in absolute terms is an approximation and greater accuracy will not be achieved by further manipulation. Uncertainty, then, will refer to variation in cost estimates from the actual costs incurred.

Factors generating uncertainty

There are numerous circumstances which contribute to uncertainty. One of the most sifnificant is changing technology. Advanced systems not only react to changes in technology but they also initiate them. This fact raises the specter of a technological barrier. Cost estimates may be predicated on the assumption of routine production when in fact critical design problems remain unsolved. Quite often this entails a reduction in the level of effectiveness necessitating a higher force level with associated higher costs. On the other hand, a technological breakthrough offers the promise of greater effectiveness which may or may not reduce costs. Specification changes of a

cost increasing nature may be the option selected rather than a reduction in costs. Operating costs are usually based on some finite time period. (Costs occurring beyond this period may be thought of as discounted at an infinite rate.) Changing technology may give rise to early obsolescence and a truncation of the expected time frame. Thus some costs may never be incurred. Conversely, delayed obsolescence, or the failure to program for a replacement system, will extend the time horizon and the period over which costs are incurred.

Uncertainty is also generated by enemy actions. An increase in his force level may cause a revision of our force level. Similarly his introduction of an advanced system may signal the premature obsolescence of one of our systems. The initiation of war is an extreme case of enemy action which affects a system in two ways. First, the return on a weapon system may be considered as being higher in the event of war. Secondly, this system will be consumed earlier with a sudden suspension of operating costs. This is equivalent to a high discount rate. Generally speaking, then, systems costs should be more heavily discounted, to make them more acceptable, when the probability of war is high.

All of the above considerations may be loosely grouped into a category known as requirements uncertainty. Fisher has concluded on the basis of empirical studies that this type of uncertainty causes the greatest variation in costs - sometimes by a factor of two.[1] Another source of uncertainty is that engendered by errors in basic data or by differences in cost-estimating techniques.

1. G. H. Fisher, "The Problem of Uncertainty," RAND Publication RM-3589-PR (Santa Monica: The RAND Corporation, June, 1963), p. VI-2.

Means of handling uncertainty

In a market context, allowance is made for uncertainty by adding a risk component to the discount rate. If the difference between returns and costs is positive, discounting will reduce the present value of this difference and make the investment less acceptable. In this case the returns, being higher, will be reduced more heavily than the costs. This would balance the actual occurrence of an increase in costs. A decrease in costs would simply be a bonus. A conceptual problem exists if the difference is negative. Reducing a negative difference will make the investment more acceptable. Therefore, the higher the discount rate, the more likely will a risky investment be undertaken. This is contrary to the intent of adding a risk component. However, the problem is not likely to occur in the private sector because any investment with a negative present value will not be undertaken. In the non-market context the advantage of this decision rule override does not exist. Benefits can rarely be evaluated and so only costs are discounted. As was discussed in Chapter II, costs may be discounted alone to reflect opportunity cost but it is not clear that they should be similarly discounted to account for risk. Risk discounting in a non-market context is a little more subtle than discounting a net return to hedge against increased costs. Moreover, discounting the expected cost streams cannot give us a more accurate estimate of future costs. The reason is that these costs may be greater or less than those estimated. If actual costs prove to be less than the expected value, a positive risk component would be required. If, however, actual costs exceed the expected value, a *negative* component would be required. The only way to determine

which is appropriate is to know the actual costs beforehand. On this basis, some authors feel that the solution is indeterminate and recommend that no discounting be made for risk. (They are actually recommending zero as the best estimate of the risk component.) They further contend that uncertainty should be handled by those who actually conduct the cost analysis by using modern statistical methods or Monte Carlo techniques. This is only a partial solution. To understand why this is so we must look at the type of results obtained by these methods.

Statistical analysis

Linear regression analysis[2] is a routine means of determining a cost estimating function and the variance associated with it. The unknown variable cost, denoted by Y, is expressed as a function of some independent variable X (speed, payload, range, etc.). The resulting relationship is of the form:

$$Y = \beta_o + \beta_1 X.$$

An estimate of this relationship is of the form:

$$\hat{Y} = a + b X. \qquad 3.1$$

Using the method of least squares, the sum of the squared deviations of the sample observations is minimized. That is,

$$\sum_{j=1}^{n} (Y_j - a - b X_j)^2 = \text{a minimum.}$$

Taking the partials with respect to a and b and setting them equal to

2. G. H. Fisher, "Use of Statistical Regression Analysis in Deriving Estimating Relationships," RAND Publication RM-3589-PR (Santa Monica: The RAND Corporation, June, 1963), Chapter V.

zero, we obtain the normal equations:

$$\sum_{j=1}^{n} Y_j = na + b \sum_{j=1}^{n} X_j$$

and

$$\sum_{j=1}^{n} X_j Y_j = a \sum_{j=1}^{n} X_j + b \sum_{j=1}^{n} X_j^2 .$$

These equations are solved for a and b to obtain the parameters of the estimating function 3.1. To determine how good an estimate has been obtained, the standard error S is computed by

$$S = \sqrt{\frac{\Sigma(Y_j - Y_p)^2}{n - 2}}$$

where Y_j is an actual observed value of the sample and Y_p is the predicted value obtained by using 3.1. Two degrees of freedom are lost because of the two parameters, a and b, already estimated. However, the analyst is more concerned with predicting future costs in a population which his sample represents not in the variance of his sample. He will usually construct a confidence interval by adding to and subtracting from his estimate the following value:

$$S' = S\, t_{1-\alpha/2} \sqrt{\frac{n}{n+2}\left[\frac{n+1}{n} + \frac{(X^* - \bar{X})^2}{\Sigma(X_j - \bar{X})^2}\right]}$$

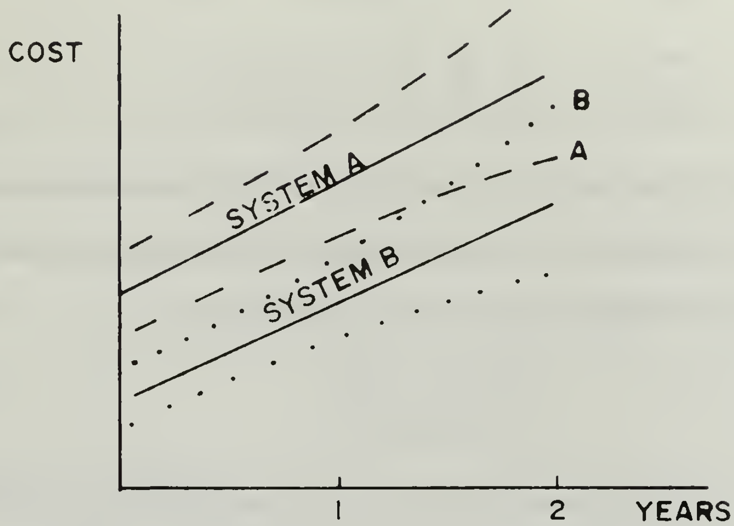
where $t_{1-\alpha/2}$ = the value from the t-distribution corresponding to the significance level α.

and X^* = the value of the independent variable which will determine the future cost.

The above assumes that for a given X, the Y's are normally distributed about a mean $Y = \beta_o + \beta_1 X$.

For fixed X, the length of this confidence interval will increase as costs are projected further into the future and values of X^* are further removed from $\bar{X}$. As depicted in Figure 1, the uncertainty increases over time. As the confidence intervals for two alternate

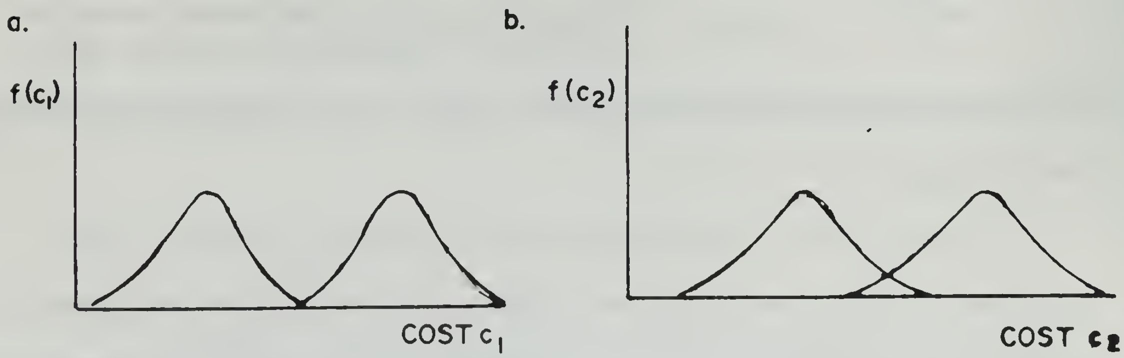
systems' costs begin to overlap, we become more uncertain as to which system will have the lower cost.



ESTIMATE OF COSTS OVER TIME
FIGURE 1

In year 1, any cost estimate for System A within its confidence interval is greater than any estimate contained within the interval for System B. In year 2, however, System B may have a cost (point B) higher than that of System A (point A).

We may proceed with our assumption that the cost estimates are normally distributed over the respective regression lines. The situation in the first year may now be depicted as in Figure 2a and that in the second year as in Figure 2b.



ASSUMED PROBABILITY DISTRIBUTIONS FOR COSTS
FIGURE 2

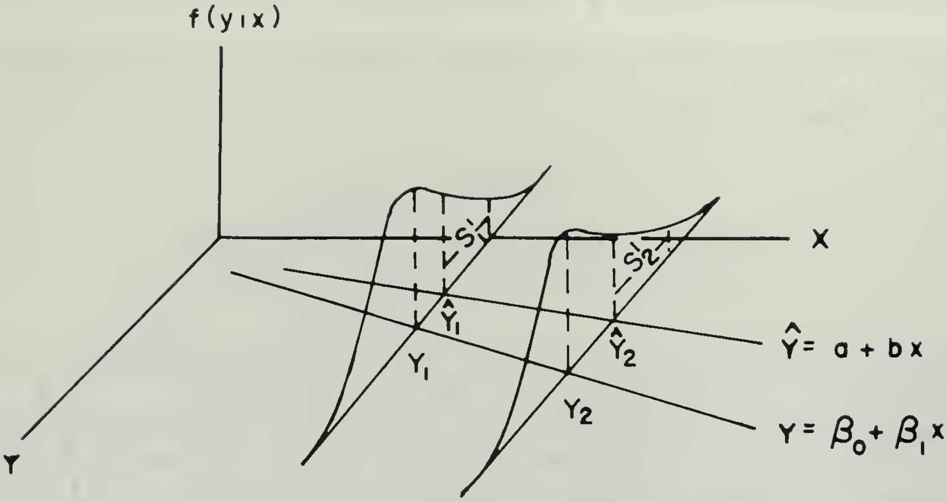
As a specific example assume that the estimating functions for System A and System B were determined from samples of size 20 and 10 respectively and the confidence intervals for each in the two years are as shown in Table 1.

TABLE 1

SYSTEMS COSTS IN MILLIONS OF DOLLARS

SYSTEM	A	B	A	B
UPPER LIMIT	875	810	1420	1820
EXPECTED VALUE	750	700	1000	1150
LOWER LIMIT	625	590	580	480
YEAR	1		2	

In the first year the expected cost of System A is 50 million more than System B but in the second year, it is 150 million less. In other words, the difference between the expected costs in the second year is of greater magnitude than that for the first year but there is considerably more uncertainty about the former. How, then, does an analyst treat the two differences to determine which system has the lower total expected costs? Clearly he does not want to treat the cost differences equally. He wants to reduce the impact of the cost difference which is more uncertain. This is the intent of discounting for risk, not manipulation of the costs to make them more accurate relatively. The crux of the problem is to make the difference between the expected values equally significant. Statistical theory does not provide us with any means of accomplishing this. The problem is that the future expected costs $\hat{Y}_1$, and $\hat{Y}_2$ are actually random variables in the normal population about the true regression line as shown in Figure 3.



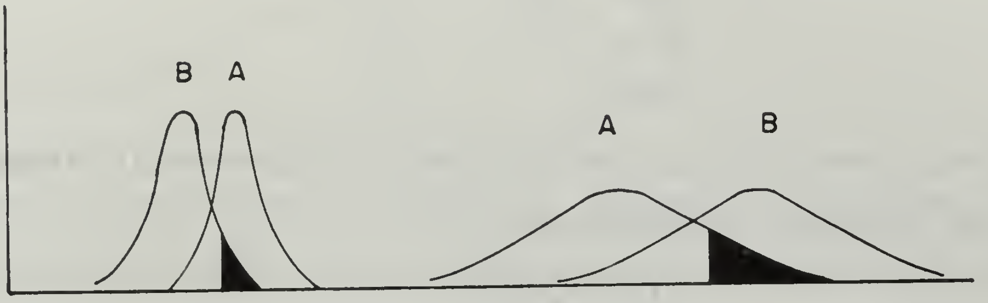
DISTRIBUTION OVER THE REGRESSION LINE

FIGURE 3

For given X, no probability distribution can be established about the random variable $\hat{Y}$. The only meaningful distribution is about the population mean Y. Moreover, the changing length of the confidence interval $\hat{Y} \pm S'$ does not imply a changing population. The variance of the true population remains the same. This is an underlying assumption of the regression model. Although the confidence intervals for the expected costs of the two systems overlap, we cannot perform any hypothesis tests for the difference of $\hat{Y}_A - \hat{Y}_B$. Any attempt to compute a numerical value for the risk component is arbitrary and a departure from statistical theory.

Nevertheless, we are still faced with a decision-making problem. Institutionally, linear regression models are not the sole source of cost estimates. One cost estimating approach may be used to estimate procurement costs while others are used to estimate various operating

costs. Therefore, the homoscedasticity assumption is tenuous and should be dropped. The variances of the cost estimates for System A and System B appear to change over time as shown in Figure 4.



ASSUMED DISTRIBUTION OF COST ESTIMATES
FIGURE 4

We will proceed in an ad hoc manner and assume that the variance is equal to $S^2 = \left[S'/t_{1-\alpha/2}\right]^2$. For our example $S_A = 60$ and $S_B = 50$ for the first year and $S_A = 200$ and $S_B = 300$ for the second year.

In hypothesis testing we usually select a significance level α and determine if there is any significant difference between two sample means. In this case, we will assume that the difference in expected costs for any year is significant and determine the level of significance. The procedure will be to compute a statistic

$$t' = \frac{\hat{Y}_A - \hat{Y}_B}{\left[\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}\right]^{1/2}}$$

and compare it with

$$t = \frac{\frac{S_A^2}{n_A} t_{1-\alpha/2}(n_A-1) + \frac{S_B^2}{n_B} t_{1-\alpha/2}(n_B-1)}{\frac{S_A^2}{n_A} + \frac{S_B^2}{n_B}}$$

The lowest value of α for which $t' > t$ will be the significance level.

For our example, in the first year

$$\hat{Y}_A - \hat{Y}_B = 50$$

and

$$t' = \frac{50}{\left[\frac{(60)^2}{20} \frac{(50)^2}{10}\right]^{1/2}} = 2.415 \quad .$$

$$t = \begin{cases} 2.191 & \text{at} \quad \alpha = .05 \\ 3.087 & \qquad\quad .01 \end{cases} \quad .$$

Therefore, the first year cost estimates are significantly different at the .05 level. For the second year

$$\hat{Y}_A - \hat{Y}_B = 150$$

and

$$t' = \frac{150}{\left[\frac{(200)^2}{20} + \frac{(300)^2}{10}\right]^{1/2}} = 1.428 \quad .$$

$$t = \begin{cases} 2.231 & \text{at} \quad \alpha = .05 \\ 1.338 & \qquad\quad .20 \end{cases}$$

The second year cost estimates are significantly different at the .20 level. These significance levels are represented by the shaded areas shown in Figure 4.

In order to treat the differences in cost estimates with the same degree of uncertainty we would like the significance level for the second year to equal that of the first year. Although there are no means of actually accomplishing this, there are reduced variances

$$\tilde{S}_A^2 = c^2 S_A^2$$

and

$$\tilde{S}_B^2 = c^2 S_B^2$$

where

$$c = \frac{t'}{t_{.95}} = .64$$

such that the significance level for the second year would be .05. By changing the variances we also change the expected values so that

$$\tilde{Y}_A = c\hat{Y}_A$$

and

$$\tilde{Y}_B = c\hat{Y}_B \quad .$$

The adjusted difference in our cost estimates for the second year is

$$c(\hat{Y}_B - \hat{Y}_A) = .64(150) = 96 \quad .$$

This constant c is actually our discount factor and is equivalent to

$$c = (1+r)^{-2} \quad .$$

Solving for r,

$$r = (.64)^{-\frac{1}{2}} - 1 = .25 \quad .$$

Thus, for the example given, the risk component of the discount rate is approximately 25 percent. This appears to be a rather high rate but the example reflected an extreme case which was characterized by large variances and in which differences in expected values changed radically.

One point that should be emphasized is the fact that there is no unique value for the risk component as there is for the normative component. The former is a function of the degree of uncertainty surrounding a particular set of cost estimates. Some estimates, such as those dealing with advanced or complicated systems, will have considerable uncertainty surrounding them and the risk component will be relatively high. Estimates of other types of equipment such as vehicles or individual weapons will reflect much less uncertainty, particularly if more historical data is available for making these estimates (the sample sizes are larger). For routine items of equipment, the change in the confidence intervals will be smaller and the levels at which expected costs are significantly different will be more nearly equal. Therefore, the risk component will be correspondingly smaller. Since the risk component is a variable whereas a unique estimate has been obtained for the normative component, the two should be used separately rather than combined into one rate when discounting costs.

In the model presented, the use of a single independent variable in equation 3.1 would only reflect the uncertainty caused by the sample size or by errors in basic data. It has been stated previously that changing technology and changes in specifications cause much of the uncertainty regarding systems' costs. It would be extremely difficult, even in retrospect, to determine how much of the difference between the estimated cost and the actual cost was due to these factors. However, a simple way of accounting for these factors is to add time as another independent variable. Presumably with a longer development period, changes in technology would exert a greater impact and there would be more specification changes. Our estimating function becomes

$$\hat{Y} = a + bX + cT.$$

There are some changes in that we now have three normal equations to solve for the parameters a, b, and c and we lose three degrees of freedom in estimating these parameters. The equations for computing confidence intervals are also modified but the rest of the model remains the same.

Uncertainty due to the probability of war cannot be similarly incorporated into the model because the historical data used in determining the estimating function would not account for the probability of war. Any method of allowing for this probability appears to be arbitrary. Nevertheless, one straightforward means of doing so is to consider the probability of actually incurring the estimated costs for each year. Assuming the probability of war in each year is .10, the expected total cost

$$E[C] = C_o + .9\ C_1 + .81\ C_2 + \ldots + (.9)^n C_n\ .$$

The risk discount rate in this case is the cumulative probability of war not occurring. This rather simple model is only a first approximation and is based on a number of simplifying assumptions. For a more rigorous treatment, the following factors should be considered. First, the probability of war in year t is higher if war has occurred in year t-1 since wars usually continue beyond one year. This suggests that a Markov model would be more appropriate. Secondly, the conditional probability of a system being destroyed, given that war has occurred, is a function of the intensity and duration of the war. The estimation of these probabilities is not as hopeless as it may appear. Many strategic systems, such as the Minuteman or Polaris, will be used only in the event of a general war and the probability of this occurring is presumably small. In other words, we can ignore the probability of an insurgency or limited war when talking about such systems. Furthermore, the number of these systems to be employed can be specified in contingency plans. Conversely, other systems, such as tactical aircraft, will be employed over the whole spectrum of war, giving us a high probability of commitment. There is also empirical evidence to suggest that aircraft attrition rates will be kept within rather narrow limits. High attrition rates in a particular sector may cause the curtailment of air operations in this sector. The implication is that the destruction of a particular system is not a completely uncontrolled variable and the probability of destruction can be narrowly defined. In view of the above, a general model for treating the probability of war would be arbitrary and hedged with numerous assumptions. The use of models associated with specific systems, and based on empirical data, is therefore recommended.

The procedure for determining the total risk component may be summarized as follows. Select a suitable cost estimation model which includes changing technology or time as an independent variable. Although a linear regression model was used for illustration, curvilinear regression models, Monte Carlo simulations, or PERT may be more appropriate. Using this model, determine the expected cost of each system for each year and compute the associated confidence intervals. Determine, also, the significance level for the cost difference in the first year (or arbitrarily select a significance level) and find the constant c which would change the probability distributions such that all cost differences are equally significant. Multiply the expected costs by this constant. The probability of war should be treated separately and in the context of the specific systems under consideration. Although the discounted estimates will presumably be less than actual costs, and therefore appear to be less accurate when viewed in an absolute sense, the relationship between two different estimates will be more meaningful. The difference between cost streams becomes smaller the more heavily we discount in the same sense as the net returns become smaller by discounting in the market context.

CHAPTER IV

SUMMARY

Resources are not free. There is either an explicit cost of acquiring them or an opportunity cost of sacrificing alternate uses of them. In the public sector the real cost of acquiring capital is not as clearly defined as it is in the private sector. The different options of financing public investments obscure the cost of capital. One estimate of this cost was shown to be the government bond rate of 4.75 percent. Most public investments pre-empt resources in the private sector causing curtailment of capital investment and deferred growth. The marginal rate of return in the private sector may be considered as the opportunity cost of public investments. Accordingly, the marginal rate of return in the private sector was also proposed as an estimate of the interest rate. This is intuitively appealing since the marginal return in both sectors should be equal for an efficient allocation of resources between both sectors. Furthermore, the marginal return should equal the marginal cost of an investment project. Because of the difficulty in measuring the marginal rate of return in the private sector, an estimate had to be derived from the average rate of return. The average rate was determined empirically to be 16.5 percent. A maximum estimate of the marginal rate in the neighborhood of 10 percent appears reasonable. It was concluded that the interest rate in the public sector lies between 4.75 and 10 percent. Sensitivity analysis over this range will be sufficient to evaluate many public investments. If a more precise estimate of the interest rate is needed, an average of the two estimates, 7 or 8 percent, is recommended.

The interest rate is not the only factor to be considered in determining the discount rate. Because of the stochastic properties of investments a risk factor must also be considered. This risk factor is not unique due to the different degrees of risk associated with various types of investments. Discounting for risk in the private sector has the effect of reducing a net return to allow for increased costs or smaller returns. In the public sector returns are rarely measured and so only costs can be discounted. It would appear that discounting costs alone would make all investments more acceptable, a concept which is contrary to the intent of discounting for risk. Actually, it is the variance in cost estimates which we would like to reduce, not simply the costs themselves. The difference between the expected costs of two alternate investments changes over time and becomes more uncertain. We would like to treat the cost difference in each time frame with equal significance. In hypothesis testing, a significance level is chosen and the difference between two expected values is considered significant or not significant at the level chosen. A reverse procedure was recommended whereby the difference in each time frame was considered significant and the level of significance was determined. The cost streams could then be multiplied by a derived vector to make the differences in expected costs equally significant. The elements of this vector may be interpreted as the risk discount rates for each time frame.

Technological change may be treated endogenously by including time as an independent variable in the cost estimating model. The probability of war, however, should be treated separately. Once the cost stream has been discounted for opportunity costs and then for

risk, it should be multiplied by the complementary probabilities for each time period. The sum of these terms will yield the expected value of the cost stream in today's dollars. In summary, there is no unique discount rate. Each investment must be considered in relation to its alternate. For a least cost criterion, the cost stream with the lowest expected value will determine the preferred investment.

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APPENDIX A

AVERAGE RATES OF RETURN IN THE PRIVATE AND PUBLIC SECTORS

A question that might be raised is: Can a relationship be inferred between the average rate of return for the private sector and that of the public sector? It is not proposed that average rates of return be equated. If there is a choice between two investment opportunities which are not mutually exclusive and which reflect different internal rates of return, investment will be made in each to the extent that their marginal returns will be equal. It is axiomatic that the average rates will be different. Nevertheless, if a relationship between the average rates of return in each sector can be ascertained, then the upper bound for the marginal rate in the private sector can be mapped into one for the public sector. To illustrate how this might be accomplished, the following model is presented. It is not offered as a rigorous proof of any specific relationship; it is more of a heuristic approach to show that such a relationship exists.

<u>Assumptions</u>

1) Each sector will expend part of its budget for present consumption and invest the remainder for returns in a future time period.

2) No qualitative distinction is made between consumption in either sector or between the returns realized by either sector.

3) The marginal efficiency of capital is monetonically decreasing for each sector. Therefore, the investment opportunity curve[1] which translates present resources into future returns is concave to the origin. (See Figure 1.)

1. Suggested by Jack Hirshleifer, "An Isoquant Approach to Investment Decision Problems," <u>RAND Publication P-1158</u> (August 23, 1957), p. 5.

4) Two cases are likely

   a) The marginal efficiency of capital is the same for both sectors.

   b) The marginal efficiency of capital is different for each sector.

<u>Notation</u>

K = the GNP.

K_g = the total budget for the government sector.

$K-K_g$ = the total budget for the private sector.

P_g = the percent of the government budget spent on investment.

I_g = $K_g p_g$ = the total government investment.

r_g = the average rate of return for I_g.

p_p = the percent of the private budget spent on investment.

I_p = $(K-K_g)p_p$ = the total private investment.

r_p = the average rate of return for I_p.

C_g = $(1-p_g)K_g$ = current government consumption.

C_p = $(1-p_p)(K-K_g)$ = current private consumption.

V = the sum of future returns and present consumption for both sectors.

The objective function will be the maximization of present consumption and future returns for both sectors.

$$\begin{aligned} V &= (1 + r_g)I_g + (1 + r_p)I_p + C_g + C_p \\ &= (1 + r_g)K_g p_g + (1 + r_p)(K-K_g)p_p + (1-p_g)K_g + (1-p_p)(K-K_g). \\ &= r_g K_g p_g + r_p(K-K_g)p_p + K. \end{aligned}$$

The above function is unconstrained in that V can be made larger by taking larger values of r_g and r_p. However, assumption 3 states that as $K_g p_g$ gets larger, r_g gets smaller with a similar trade-off between $(K-K_g)p_p$ and r_p. To form an explicit constraint, r_g must be expressed

in terms of K_gp_g and r_p in terms of $(K-K_g)p_p$. A curve, as shown in Figure 1, which closely approximates a true investment opportunity curve is of the form:

$$y = \frac{2}{3}\left[K - \frac{x^2}{K}\right]$$

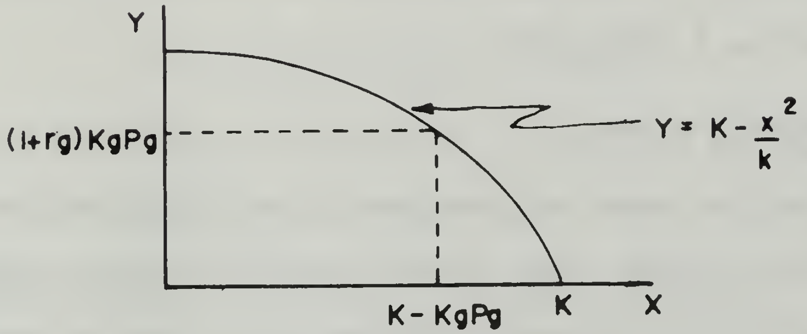
where $$x = K - K_gp_g$$

but $$y = (1 + r_g)K_gp_g \quad .$$

Therefore $$(1 + r_g)K_gp_g = \frac{2}{3}\left[K - \frac{(K - K_gp_g)^2}{K}\right]$$

or $$r_g = \frac{1}{3} - \frac{2}{3}\,\frac{K_gp_g}{K} \quad .$$

Similarly $$r_p = \frac{1}{3} - \frac{2}{3}\,\frac{(K - K_g)p_p}{K}$$



INVESTMENT OPPORTUNITY CURVE

FIGURE 1

A new function may be formed using the two constraints and two Lagrange multipliers. This function has seven unknown variables. To maximize this function, the partial derivatives with respect to each of the unknown variables will be taken and set equal to zero. The result is seven equations in the seven unknown variables

$$Z = r_gK_gp_g + r_p(K-K_g)p_p + K - \lambda_1\left\{r_g - \frac{1}{3} + \frac{2}{3}\,\frac{K_gp_g}{K}\right\} - \lambda_2\left\{r_p - \frac{1}{3} + \frac{2}{3}\,\frac{(K-K_g)p_p}{K}\right\}$$

$$\frac{\partial Z}{\partial K_g} = r_gp_g - r_pp_p - \frac{2}{3}\lambda_1\frac{p_g}{K} + \frac{2}{3}\lambda_2\frac{p_p}{K} = 0 \qquad \text{A.1}$$

$$\frac{\partial Z}{\partial r_g} = K_gp_g - \lambda_1 = 0 \quad \rightarrow \quad \lambda_1 = K_gp_g \qquad \text{A.2}$$

$$\frac{\partial Z}{\partial r_p} = (K-K_g)p_p - \lambda_2 = 0 \qquad \rightarrow \qquad \lambda_2 = (K-K_g)p_p \quad . \qquad A.3$$

$$\frac{\partial Z}{\partial \lambda_1} = - r_g + \frac{1}{3} - \frac{2}{3}\,\frac{K_g p_g}{K} = 0 \qquad \rightarrow \qquad r_g = \frac{1}{3} - \frac{2}{3}\,\frac{K_g p_g}{K} \quad . \qquad A.4$$

$$\frac{\partial Z}{\partial \lambda_2} = - r_p + \frac{1}{3} - \frac{2}{3}\,\frac{(K-K_g)}{K} p_p = 0 \qquad \rightarrow \qquad r_p \quad \frac{1}{3} - \frac{2}{3}\,\frac{(K-K_g)}{K} p_p . \qquad A.5$$

$$\frac{\partial Z}{\partial p_g} = r_g K_g - \frac{2}{3}\lambda_1 \frac{K_g}{K} = 0 \qquad \rightarrow \qquad r_g = \frac{2}{3}\,\frac{K_g p_g}{K} \quad . \qquad A.6$$

$$\frac{\partial Z}{\partial p_p} = r_p(K-K_g) - \frac{2}{3}\lambda_2 \frac{K-K_g}{K} = 0 \qquad \rightarrow \qquad r_p = \frac{2}{3}\,\frac{K-K_g}{K} p_p \quad . \qquad A.7$$

Substituting in Equation A.1,

$$r_g p_g - r_p p_p + (r_g - \tfrac{1}{3})p_g - (r_p - \tfrac{1}{3})p_p = 0 \ .$$

or

$$p_g(2r_g - \tfrac{1}{3}) = p_p(2r_p - \tfrac{1}{3}) \quad . \qquad A.8$$

Using Equations A.4 and A.6, $r_g = \frac{1}{3} - r_g \rightarrow r_g = \frac{1}{6}$.

Using Equations A.5 and A.7, $r_p = \frac{1}{3} - r_p \rightarrow r_p \quad \frac{1}{6}$.

Thus for the case where the investment opportunity curves are identical, $r_g = r_p$ for a maximum. This is not a surprising result since the average returns will be equal if the marginal returns are equal for identical investment schedules. However, if some exogenous constraint prevents the investment functions, p_g and p_p from being equal and causing a displacement away from the optimal point, the next best position which still satisfies Equation A.8 will not have the average returns equal. This, too, is not a surprising result since the marginal rates will no longer be equal. Nevertheless, the relationship between the average rates of return in the two sectors has been established by assuming a particular function for the investment opportunity curve. One interesting aspect is that r_g is relatively insensitive to large changes in the government function p_g.

Assuming $r_p = .165$

and $r_p = .25$

if $p_g = 1.0$,

then $(2r_g - \frac{1}{3}) = \frac{1}{4}(2r_p - \frac{1}{3})$

or $r_g = .1652$.

If $p_g = .5$,

then $\frac{1}{2}(2r_g - \frac{1}{3}) = \frac{1}{4}(2r_p - \frac{1}{3})$

or $r_g = .1658$.

For the case where the marginal efficiency of capital is higher for the public sector, the following function was used for r_g:

$$(1 + r_g)K_g p_g = \frac{3}{4}\left\{K - \frac{(K - K_g p_g)^2}{K}\right\}$$

Regaining the original function for r_p, Equation A.8 becomes

$$p_g(2r_g - \frac{1}{2}) = p_p(2r_p - \frac{1}{3}).$$

The optimal point is reached when the average rates are:

$$r_g = \frac{1}{4}$$

$$\text{and } r_p = \frac{1}{6} .$$

This satisfies our intuition to the extent that a more efficient investment schedule should reflect a higher average rate of return. Using the same assumptions as in the preceding case, r_g varies between .2492 and .2495 for a change in p_g from .5 to 1.0.

For the case where the marginal efficiency of capital is lower for the public sector, the following function was used for r_g:

$$(1 + r_g)K_g p_g = \frac{3}{5}\left\{K - \frac{(K - K_g p_g)^2}{K}\right\}.$$

Equation A.8 is now $p_g(2r_g - \frac{1}{5}) = p_p(2r_p - \frac{1}{3})$.

Optimal r_g is .10 and is still insensitive to a large change in p_g.

It has been shown that by assuming a particular family of functions for the marginal efficiency of capital, the average rate of return for the public sector can be inferred from an empirical return found in the private sector. It is this figure then that would be used as an upper bound on the interest rate in the public sector.

APPENDIX B

SENSITIVITY ANALYSIS

Two methods for evaluating alternative investment projects are the present value formulation and the internal rate of return rule. The former uses the cost of capital or the interest rate to discount future streams.

Letting P = the net present value,

R_t = the return in time period t,

C_t = the cost in time period t,

i = the interest rate, and

T = the lifetime of the project,

then
$$P = \sum_{t=0}^{T} (R_t - C_t)(1 + i)^{-t}. \tag{1}$$

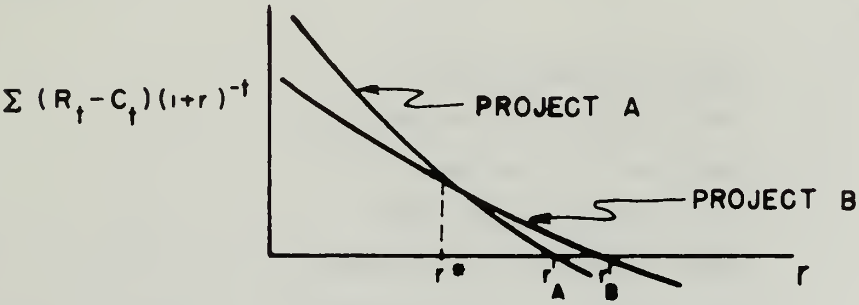
The project with the higher present value would be chosen assuming both projects are mutually exclusive. In a non-market context, however, we are uncertain as to the true value of i and hence the present value is indeterminate. The second investment criterion, however, considers the discount rate as an unknown variable and attempts to determine the discount rate which will equate the present value of future returns with that of the associated cost stream. This is the definition of the internal rate of return.

Letting r = the internal rate of return,

then
$$\sum_{t=0}^{T} R_t(1 + r)^{-t} - \sum_{t=0}^{T} C_t(1 + r)^{-t} = 0. \tag{2}$$

The procedure for solving this polynomial in r is essentially a trial and error process. If the initial estimate of r makes the left hand side of Equation 2 positive, then a higher value of r should be tried; if the result is negative, then a lower value is used. This

procedure is iterated until Equation 2 is satisfied. The investment project with the higher rate of return, however, is not necessarily the preferred one. If we plot the value of the left hand side of Equation 2 against increasing values of r, we will obtain a decreasing function. Thus for two different projects we may obtain the graph in Figure 1.



PRESENT VALUE VS. INTEREST RATES

FIGURE 1

Since the functions become zero at r_A and r_B, these are the internal rates of return for the two projects. r^* is the discount rate which makes the net returns for the two projects equivalent. In the hypothetical situation depicted, project B has the higher rate of return and would appear to be the preferred investment. Note that the functions which have been plotted are also the present value functions if r equals i. Note also that if i is less than r^*, then under the present value rule, Project A would be preferred. In other words, it is only when i is greater than r^* that the internal rate of return yields the same result as the present value rule. We can use this information in making a sensitivity analysis.

For example, let us suppose that we are presented with two weapon systems of equal effectiveness and with cost streams as depicted in Table 1.

TABLE 1

	INITIAL COST	ANNUAL OPERATING COSTS
SYSTEM A	500 MILLION	50 MILLION
SYSTEM B	280 "	100 "

*5 year expected life span

The discount rate which make these two cost streams equivalent is approximately 4.5 percent. If we are confident that the true discount rate is greater than this figure, then System B is the preferred system. Thus, in this example, we did not have to know the precise value of the true discount rate. We merely required a lower bound on it.

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13. ABSTRACT

Systems Analysis uses many disciplines and sophisticated techniques to determine the relative effectiveness of alternate systems. This relationship can then be reversed by an arbitrary choice of a discount rate. In the private sector, the discount rate is determined by the cost of capital. In the public sector, the cost of acquiring capital is not clearly defined. The following proposals for the interest rate in the public sector are considered: (1) the government bond rate; (2) the rate of growth of the national product; and (3) a rate derived from the average rate of return in the private sector. It is concluded that the interest rate in the public sector currently lies between 4.75 and 10 percent.

The circumstances which generate uncertainty and the means of handling this uncertainty are also discussed. A procedure is recommended which modifies the difference between expected costs to make them equally significant. Particular attention is focused on uncertainty occasioned by changing technology and the probability of war. It is concluded that a unique estimate of the risk component is indeterminate and that uncertainty should be considered in the context of the specific systems under consideration.

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